Practice Exercises For Mathematics Placement Test - Test 2
(Corresponds to Precalculus Competency - Preparedness for M151)

The Test 2 Placement exam is a multiple choice exam covering topics typically found in a Precalculus course. Passing the exam means that you are prepared to take M151 - Calculus I. Below are practice exercises to review some of the material required for the exam. Answers to the practice exercises can be found on the last pages.

Major Topics: A student taking the exam should be prepared to

- Factor and simplify polynomial, rational, radical and absolute value expressions.
- Recognize and graph equations of circles and parabolas (may need completing the square).
- Solve equations and inequalities involving absolute values, polynomial, and rational expressions.
- Work with function notation and function operations (including composition and the difference quotient).
- Work with quadratic and rational functions, as well as inverse functions.
- Graph polynomial, rational, exponential, and logarithmic functions.
- Use the Factor Theorem, the Remainder Theorem, and polynomial division (either long division or synthetic division) to find integer, rational, and irrational roots of polynomial equations.
- Solve exponential and logarithmic equations.
- Solve and graph systems of linear equations and inequalities in two or three variables.
- Know the unit-circle definition of trigonometric functions.
- Know right triangle trigonometry (opposite, adjacent, and hypotenuse).
- Evaluate trigonometric and inverse trigonometric expressions.
- Change radian measure to degree measure and vice versa.
- Graph trigonometric functions, including transformations.
- Know and use fundamental trigonometric identities to simplify expressions.
- Verify trigonometric identities.
- Solve trigonometric equations.

Revised October 3, 2020

1. Find all solutions to the following equations.

   (a) $x^2 + 3x - 7 = 0$ (b) $\dfrac{x^2 - 5x - 6}{x + 2} = 0$ (c) $\sqrt{4x - 3} + 2 = 7$ (d) $|x + 1| = 6$

2. Determine the set of all solutions for each, graphing inequalities on a number line.

   (a) $|x + 3| < 2$ (b) $|3x - 7| \geq 1$ (c) $(x - 3)(x + 1) \leq 0$

   (d) $(2x - 5)(x + 3) > 0$ (e) $\dfrac{(x + 2)(x - 1)}{x - 3} \leq 0$ (f) $\dfrac{1}{x + 1} > \dfrac{2}{x}$

3. Graph and state the coordinates of the vertex of each.

   (a) $y = 3(x - 5)^2 + 2$ (b) $y = x^2 - 2x - 2$

4. If $f(x) = 2x^2 - x$ and $g(x) = x + 3$, evaluate each of the following.

   (a) $f(5)$ (b) $(f + g)(x)$ (c) $(fg)(x)$ (d) $(f \circ g)(x)$

   (e) $g(-3)$ (f) $(f - g)(3)$ (g) $\left(\dfrac{f}{g}\right)(1)$ (h) $(g \circ f)(4)$

   (i) $\dfrac{f(2 + h) - f(2)}{h}$ (j) $\dfrac{g(x + h) - g(x)}{h}$

5. State the domain and range of each; then graph each.

   (a) $f(x) = \sqrt{7x + 4}$ (b) $g(x) = x^3 + 1$ (c) $h(x) = |x - 2| + 1$

6. Determine the inverse function of $f(x) = \dfrac{1}{x - 1}$.

7. Graph each.

   (a) $y \leq 3x + 2$ (b) $y > x^2$

8. Find the set of solutions for each systems of equations.

   (a) $\begin{cases} 2x - y = -1 \\ x + y = 7 \end{cases}$ (b) $\begin{cases} 2x - y + z = 1 \\ x - y + 2z = 3 \\ x - y + z = 1 \end{cases}$ (c) $\begin{cases} y = x^2 + 1 \\ y = 3x + 5 \end{cases}$

9. Write an equation of the circle with center at the origin and radius of 3.

10. Determine the center and radius of the circle $(x - 1)^2 + (y + 2)^2 = 4$.

11. Determine the center and radius of the circle $x^2 + y^2 - 4y = 12$.

12. Determine whether $x - 2$ is a factor of $x^3 + 4x^2 - 2x + 4$. [Use of the Factor Theorem would be a quick way to do this.]

13. Find the quotient and remainder if $3x^4 - 2x^2 - 3x + 7$ is divided by $x + 1$. [Use of synthetic division would be a quick way to do this.]

14. Based on the coefficient of the x^4 term and the constant term, state the candidates for rational roots of $3x^4 + 2x^3 - 4x + 2 = 0$.

15. Find all real solutions (giving exact values) of $2x^4 - 2x^3 - 11x^2 - 4x + 3 = 0$.

16. Graph each.

    (a) $y = 2^x$ (b) $y = 2^{-x}$ (c) $y = \log_2 x$

17. Find all solutions for each.

    (a) $\log_2 8 = x$ (b) $\log_7 x = 0$ (c) $5^x = 125$

    (d) $4^{3x-1} = 3^{x-2}$ (e) $\log(x+1) - \log(x) = \log 4$

18. Graph each.

    (a) $f(x) = \dfrac{3x-2}{x+3}$ (b) $g(x) = \dfrac{2x}{x^2-4}$

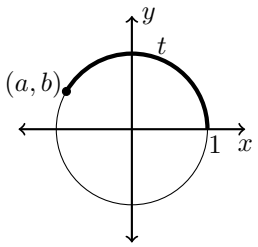
19. Find the coefficient of x^3y.

    (a) $(x+y)^4$ (b) $(3x-y)^4$

20. Use the unit-circle definition of the trigonometric functions to obtain the values of each, if t is as indicated in the figure to the right.

    (a) $\sin t$ (b) $\cos t$ (c) $\tan t$

    (d) $\cot t$ (e) $\sec t$ (f) $\csc t$



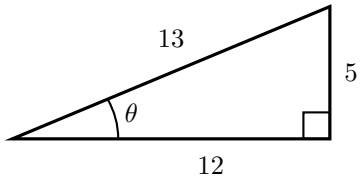
21. If $\sin\theta = -\dfrac{3}{5}$ and θ is in quadrant III, determine each.

    (a) $\cos\theta$ (b) $\tan\theta$ (c) $\cot\theta$ (d) $\sec\theta$ (e) $\csc\theta$

22. Given θ as indicated in the figure at the right, determine each of the following.

    (a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$

    (d) $\cot\theta$ (e) $\sec\theta$ (f) $\csc\theta$



23. State the values of the following.

    (a) $\sin(30°)$ (b) $\cos\left(\dfrac{\pi}{6}\right)$ (c) $\tan(30°)$ (d) $\sin\left(\dfrac{\pi}{4}\right)$ (e) $\cos(45°)$

    (f) $\tan\left(\dfrac{\pi}{4}\right)$ (g) $\sin\left(\dfrac{\pi}{3}\right)$ (h) $\cos\left(\dfrac{\pi}{3}\right)$ (i) $\tan(60°)$

24. Write each in terms of a trigonometric function of an angle in the first quadrant.

(a) $\sin(150^\circ)$ (b) $\cos(225^\circ)$ (c) $\tan(300^\circ)$

25. Change each radian measure to degree measure.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{7\pi}{6}$ (d) $\frac{3\pi}{4}$ (e) π

26. Find the exact value of $\sin^2\left(\frac{3\pi}{7}\right) + \cos^2\left(\frac{3\pi}{7}\right)$.

27. Graph each.

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = \tan x$ (d) $y = \frac{3}{2}\cos x$ (e) $y = \sin 3x$

28. Find the period and amplitude of the graph of $y = 3\cos\left(2x - \frac{\pi}{4}\right)$.

29. Reduce each to a single function of the argument θ.

(a) $\cos\theta\csc\theta$ (b) $\sec\theta - \sin\theta\tan\theta$

30. If $\sin\theta = \frac{1}{3}$ and $\cos\theta = \frac{2\sqrt{2}}{3}$, find $\sin 2\theta$.

31. If $\tan\phi = \frac{4}{3}$, find $\cos 2\phi$.

32. Find the set of solutions in the interval $0 \le x \le 2\pi$ for each equation.

(a) $\sin x = \frac{1}{2}$ (b) $2\cos x + 1 = 2$ (c) $\tan x = 1$

(d) $\sin 2x + 1 = 0$ (e) $2\cos^2 x - \sin x = 1$ (f) $\tan^2 x - 3 = 0$

33. Let $\angle C$ be the right angle of triangle $\triangle ABC$.

(a) If the length of the side opposite $\angle A$ is 6 centimeters and the measure of $\angle B$ is 60°, find the length of the hypotenuse.

(b) If the measure of $\angle A$ is 45° and the length of the side opposite $\angle B$ is 25 feet, find the length of the side opposite $\angle A$.

34. Find the value of each.

(a) $\sin^{-1}\left(-\frac{1}{2}\right)$ (b) $\cos(\tan^{-1}\sqrt{3})$

You should know the following identities.

Logarithmic Identities

$$\log(AB) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\log(A^n) = n \log A$$

Exponent Identities

$$A^m A^n = A^{m+n}$$

$$\frac{A^m}{A^n} = A^{m-n}$$

$$(A^m)^n = A^{mn}$$

$$(AB)^n = A^n B^n$$

$$\left(\frac{A}{B}\right)^n = \frac{A^n}{B^n}$$

Trigonometric Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin(-x) = -\sin x$$

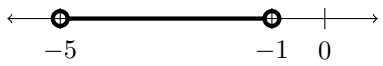
$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

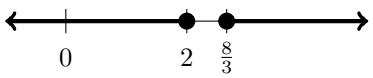
Answers For Test 2

1. (a) $x = \dfrac{-3 \pm \sqrt{37}}{2} \approx 1.54, -4.54$ (b) $x = 6, -1$ (c) $x = 7$ (d) $x = 5, -7$

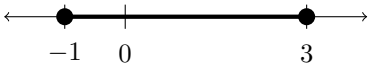
2. (a) $-5 < x < -1$



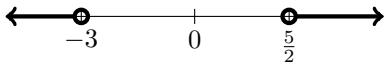
(b) $x \leq 2$ or $x \geq 8/3$



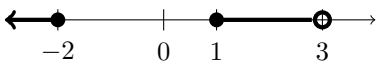
(c) $-1 \leq x \leq 3$



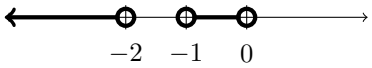
(d) $x < -3$ or $x > 5/2$



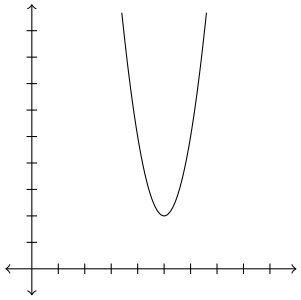
(e) $x \leq -2$ or $1 \leq x < 3$



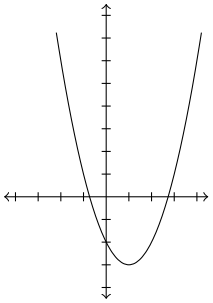
(f) $x < -2$ or $-1 < x < 0$



3. (a) Vertex $(5, 2)$



(b) Vertex $(1, -3)$



4. (a) 45 (b) $2x^2 + 3$ (c) $2x^3 + 5x^2 - 3x$ (d) $2(x+3)^2 - (x+3) = 2x^2 + 11x + 15$

   (e) 0 (f) 9 (g) $\dfrac{1}{4}$ (h) 31 (i) $7 + 2h$ (j) 1

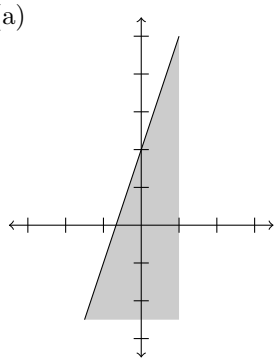
5. (a) Domain: $[-4/7, \infty)$, Range: $[0, \infty)$

   (b) Domain: all real numbers ($\mathbb{R}$), Range: all real numbers ($\mathbb{R}$)

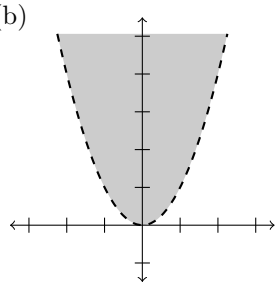
   (c) Domain: all real numbers ($\mathbb{R}$), Range: $[1, \infty)$

6. $f^{-1}(x) = 1 + \dfrac{1}{x}$

7. (a)



(b)



8. (a) $x = 2$, $y = 5$ (b) $x = 0$, $y = 1$, $z = 2$ (c) $x = 4, y = 17$ and $x = -1, y = 2$

9. $x^2 + y^2 = 9$

10. Center $(1, -2)$, radius 2

11. Center $(0, 2)$, radius 4

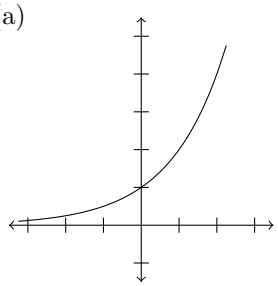
12. Not a factor.

13. Quotient: $3x^3 - 3x^2 + x - 4$, Remainder: 11

14. $\pm 2, \pm\frac{2}{3} \pm 1, \pm\frac{1}{3}$

15. $x = -1, 3, \dfrac{-2 \pm \sqrt{12}}{4} = \dfrac{-1 \pm \sqrt{3}}{2}$

16. (a)



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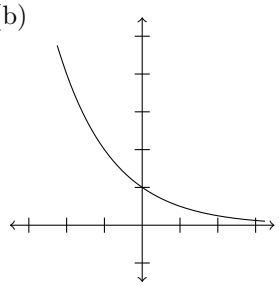

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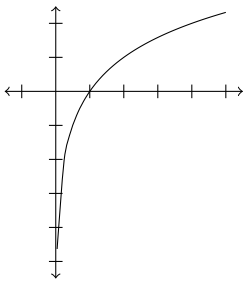
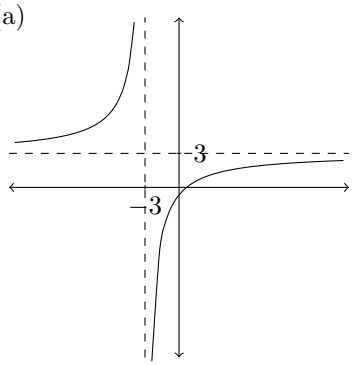


17. (a) $x = 3$ (b) $x = 1$ (c) $x = 3$ (d) $x = \dfrac{\ln 4 - 2\ln 3}{3\ln 4 - \ln 3} = \dfrac{\ln \frac{4}{9}}{\ln \frac{64}{3}}$ (e) $x = \dfrac{1}{3}$

18. (a)



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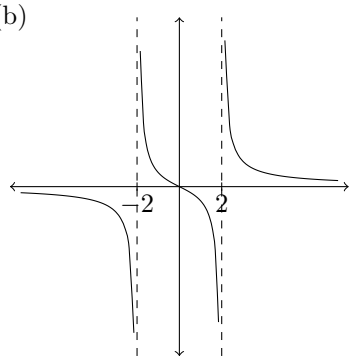


19. (a) 4 (b) -108

20. (a) b (b) a (c) $\dfrac{b}{a}$ (d) $\dfrac{a}{b}$ (e) $\dfrac{1}{a}$ (f) $\dfrac{1}{b}$

21. (a) $-\dfrac{4}{5}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{4}{3}$ (d) $-\dfrac{5}{4}$ (e) $-\dfrac{5}{3}$

22. (a) $\dfrac{5}{13}$ (b) $\dfrac{12}{13}$ (c) $\dfrac{5}{12}$ (d) $\dfrac{12}{5}$ (e) $\dfrac{13}{12}$ (f) $\dfrac{13}{5}$

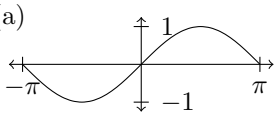
23. (a) $\dfrac{1}{2}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{\sqrt{3}}{3}$ (d) $\dfrac{\sqrt{2}}{2}$ (e) $\dfrac{\sqrt{2}}{2}$ (f) 1 (g) $\dfrac{\sqrt{3}}{2}$ (h) $\dfrac{1}{2}$ (i) $\sqrt{3}$

24. (a) $\sin 30°$ (b) $-\cos 45°$ (c) $-\tan 60°$

25. (a) 60° (b) 90° (c) 210° (d) 135° (e) 180°

26. 1

27. (a)



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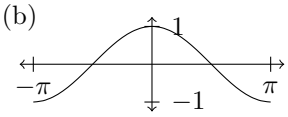

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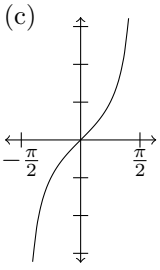

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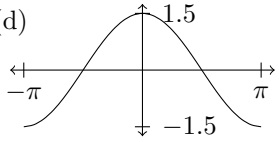

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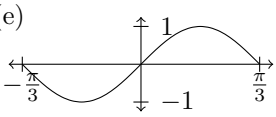

28. Amplitude = 3, Period = π

29. (a) $\cot\theta$ (b) $\cos\theta$

30. $\dfrac{4\sqrt{2}}{9}$

31. $-\frac{7}{25}$

32. (a) $\frac{\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{4}, \frac{5\pi}{4}$ (d) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (e) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ (f) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

33. (a) 12 cm (b) 25 ft

34. (a) $-\frac{\pi}{6}$ (b) $\frac{1}{2}$